OCR
Oxford Cambridge and RSA

# Wednesday 7 June 2017 – Morning

## A2 GCE MATHEMATICS

**4730/01** Mechanics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4730/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

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## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

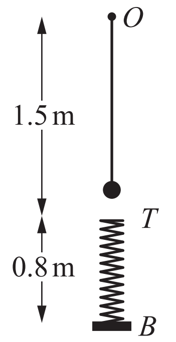
## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**1** A particle of mass 0.2 kg is moving with speed $4\,\text{m}\,\text{s}^{-1}$ in a straight line on a smooth horizontal plane. A horizontal impulse of magnitude 1.2 N s acts on the particle. After the impulse acts the particle is moving with speed $5\,\text{m}\,\text{s}^{-1}$.

Find the angle between the directions of motion of the particle before and after the impulse acts. Find also the angle between the direction in which the impulse acts and the initial direction of motion of the particle. **[5]**

**2**




A particle of mass $m$ kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $24mg$ N. The other end of the string is attached to a fixed point $O$. Directly beneath $O$ there is a light elastic spring, of natural length 0.8 m and modulus of elasticity $32mg$ N. The bottom of the spring is attached to a fixed point $B$ and the top of the spring is at a point $T$, 1.5 m vertically below $O$; the spring is constrained to remain vertical. The particle is projected from $O$ with speed $0.7\,\text{m}\,\text{s}^{-1}$ vertically downwards. When the particle reaches $T$ it becomes attached to the spring and it remains attached to the spring throughout the subsequent motion. The diagram shows the position as the particle first approaches $T$.

**(i)** Show that the speed of the particle at the instant when it becomes attached to the spring is $3.5\,\text{m}\,\text{s}^{-1}$. **[3]**

**(ii)** Find the distances below $O$ at which the particle is instantaneously at rest in the subsequent motion. **[5]**

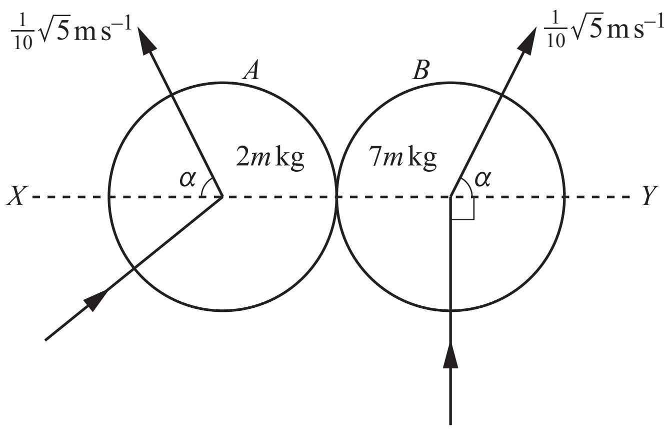
**3** A particle of mass 0.2 kg travels in a straight line on a smooth horizontal surface. At time $t$ seconds it is $x$ m from a fixed point $O$ and is moving away from $O$ with velocity $v\,\text{m}\,\text{s}^{-1}$. A force of magnitude $\frac{1}{2}\left(12-\frac{1}{4}v\right)^{\frac{1}{2}}$ N acts on the particle in the direction of motion. At time $t = 0$ the particle is at $O$ and has velocity $12\,\text{m}\,\text{s}^{-1}$.

**(i)** State the maximum possible velocity of the particle. **[1]**

**(ii)** Find an expression for $v$ in terms of $t$, valid while the particle is accelerating. **[6]**

**(iii)** Hence find the distance travelled by the particle as its velocity increases from $12\,\text{m}\,\text{s}^{-1}$ to $32\,\text{m}\,\text{s}^{-1}$. **[4]**

**4**




Two uniform smooth spheres, $A$ and $B$, of equal radius, with masses $2m$ kg and $7m$ kg respectively, collide on a horizontal surface. Immediately before the collision $B$ is moving in a direction perpendicular to the line of centres, $XY$. Immediately after the collision both $A$ and $B$ are moving with speed $\frac{1}{10}\sqrt{5}\,\mathrm{m\,s^{-1}}$; they are moving on the same side of $XY$ and are both moving in directions making an angle $\alpha$ with $XY$, where $\tan\alpha = 2$ (see diagram).

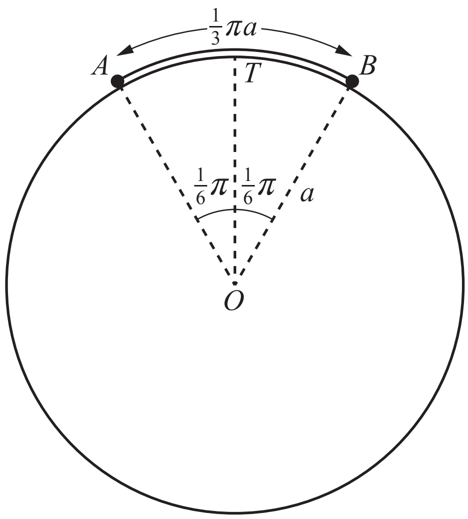
**(i)** Find the speed and direction of motion of $A$ before the collision with $B$. Find also the coefficient of restitution in the collision. **[7]**

Subsequently another uniform smooth sphere, $C$, of mass $3m$ kg and with the same radius as $A$ and $B$, collides with $A$. The line of centres of the collision between $C$ and $A$ is parallel to $XY$; immediately before this collision $C$ is travelling with speed $U\,\mathrm{m\,s^{-1}}$ from left to right parallel to $XY$. This collision is perfectly elastic.

**(ii)** Explain why $A$ will have a second collision with $B$ if $U$ is large enough. **[1]**

**(iii)** Find the greatest value of $U$ for which $A$ will **not** subsequently have a second collision with $B$. **[6]**

**5**



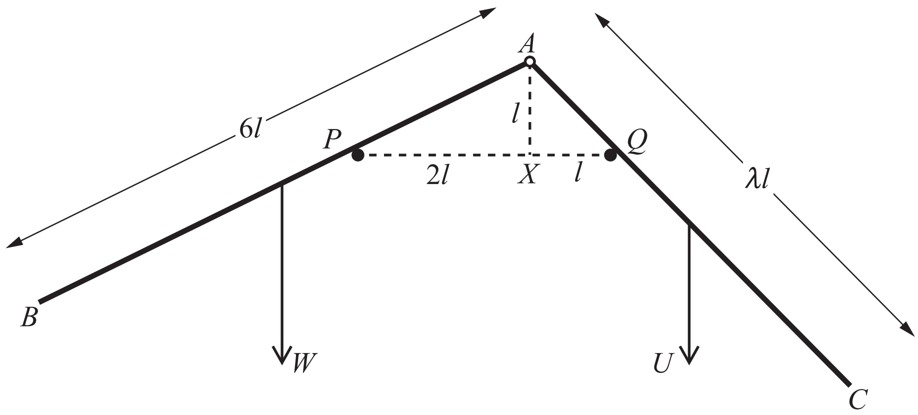

Particle $A$ of mass $3m$ and particle $B$ of mass $2m$ are joined by a light inextensible string of length $\frac{1}{3}\pi a$. The particles and the string rest in a vertical plane on the surface of a smooth cylinder, of radius $a$, which has its axis horizontal; $O$ is the centre of the vertical cross-section of the cylinder containing the particles and $T$ is the uppermost point on the surface of the cylinder. The string is taut and radii $OA$ and $OB$ each make an angle of $\frac{1}{6}\pi$ radians with $OT$ (see diagram). $A$ and $B$ are released from rest, and after $t$ seconds angle $AOT$ is $\left(\frac{1}{6}\pi + \theta\right)$ radians and both $A$ and $B$ are moving with speed $v$.

**(i)** Show that, while $A$ and $B$ remain in contact with the cylinder,

$$v^2 = \tfrac{2}{5}ag\left(5\cos\tfrac{1}{6}\pi - 3\cos\left(\tfrac{1}{6}\pi + \theta\right) - 2\cos\left(\tfrac{1}{6}\pi - \theta\right)\right).$$ **[4]**

**(ii)** Find an expression in terms of $m$ and $g$ for the force exerted on $A$ by the cylinder at the instant when $B$ is passing through $T$. **[4]**

**6**

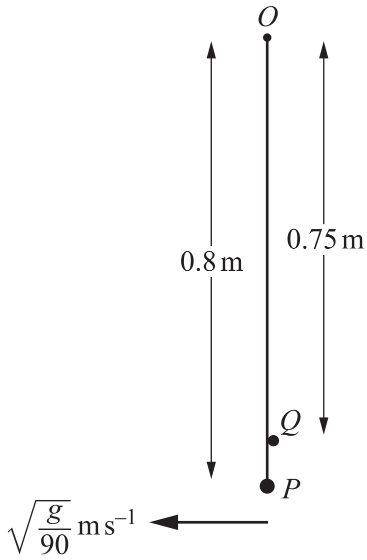



Two uniform rods, $AB$ and $AC$, are freely jointed at $A$. The weight of $AB$ is $W$ and its length is $6l$; the weight of $AC$ is $U$ and its length is $\lambda l$, where $\lambda$ is a constant. The rods rest in equilibrium in a vertical plane on two small smooth pegs, $P$ and $Q$, which are a distance $3l$ apart at the same horizontal level. The point $X$ is a distance $l$ vertically below $A$ and lies on the line joining $P$ and $Q$ such that $PX = 2l$ and $XQ = l$ (see diagram).

**(i)** Show that the magnitude of the force acting on the rod $AB$ at $P$ is $1.2W$, and express the magnitude of the force acting on the rod $AC$ at $Q$ in terms of $\lambda$ and $U$. **[4]**

**(ii)** Find the value of the constant $k$ for which $U = kW$. Find also the value of $\lambda$. **[7]**

**7**




A light inextensible string of length 0.8 m is attached to a fixed point $O$. A particle $P$ of mass $m$ kg hangs in equilibrium attached to the lower end of the string. A small peg $Q$ is fixed at a distance 0.75 m vertically below $O$. The particle $P$ is given a horizontal velocity of $\sqrt{\frac{g}{90}}$ m s$^{-1}$ so that the string initially moves to the left, away from the peg $Q$ (see diagram).

**(i)** Prove that the motion of $P$ while it is to the left of its initial position is approximately simple harmonic, and find the period of this motion. **[7]**

**(ii)** Find the time that elapses between the first and second occasions that the string makes an angle of 5° with the vertical. Find also the linear speed of $P$ in this position. **[5]**

**(iii)** When the string returns to the vertical position, $P$ begins to move in a circle with centre $Q$. Explain with a reason whether the motion of $P$ is approximately simple harmonic motion while $P$ is to the right of its initial position. **[3]**

**END OF QUESTION PAPER**

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